

Preserving universal resources for one-way quantum computing

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The common spin Hamiltonians such as the Ising, XY , or Heisenberg model do not have ground states that are the graph states needed in measurement-based quantum computation. Various highly-entangled many-body states have been suggested as a universal resource for this type of computation, however, it is not easy to preserve these states in solid-state systems due to their short coherence times. Here we propose a scheme for generating a Hamiltonian that has a cluster state as ground state. Our approach employs a series of pulse sequences inspired by established NMR techniques and holds promise for applications in many areas of quantum information processing.

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Measurement-based quantum computation (MQC) is a new computing paradigm [1]. Of particular interest are universal resources of one-way quantum computation, a MQC scheme that requires only local measurements [2]. In the original scheme of one-way quantum computing, one initially creates a many-qubit cluster state by applying phase gates or equivalent gate operations which can be realized using the Ising interaction between qubits. Many promising methods to generate cluster states using solid-state qubits have been proposed [3, 4]. However, these states are not the ground states of spin Hamiltonians with typical qubit-qubit interactions of Ising, XY , and Heisenberg types [5].

One of the established universal resources are two-dimensional (2D) cluster states. Another promising candidates is the Affleck-Kennedy-Lieb-Tasaki (AKLT) state on the honeycomb lattice [6], a resonance valence bond (RVB) type state which is a special projected entangled pair state (PEPS) [7, 8]. Yet, the AKLT state requires non-trivial Hamiltonians with spin greater than $1/2$, which are not easy to realize in solid-state systems.

In this Letter, we present a new method of constructing stabilizer Hamiltonians whose ground states are the desired cluster states. Our approach relies on manipulating a two-body Hamiltonian using pulse-sequence techniques developed in the nuclear magnetic resonance (NMR) context [9, 10]. We show that, starting from the Ising and XY models, one can induce an effective dynamics described by a stabilizer Hamiltonian [2]

$$H_{\text{stab}} = - \sum_i K_i, \quad (1)$$

where $K_i = \sigma_i^x \bigotimes_{j \in \text{nbhd}(i)} \sigma_j^z$ are the correlation operators and the direct product runs over all nearest neighbors of the lattice site i (σ_i^x and σ_j^z are the Pauli matrices). Thus, the main idea of this paper is to generate a stabilizer Hamiltonian by transforming the original Hamiltonian.

We assume the original Hamiltonian to be of the form

$H = H_0 + H_{\text{int}}$ where

$$H_0 = \sum_i (\Omega_i \sigma_i^x + \varepsilon_i \sigma_i^z) \quad (2)$$

is a single-qubit part and H_{int} the interaction part. We take H_{int} to be of Ising $H_{\text{Ising}} = \sum_{i < j} J_{ij} \sigma_i^z \sigma_j^z$, XY $H_{XY} = \sum_{i < j} J_{ij} [XY]_{ij}$, and Heisenberg form $H_H = \sum_{i < j} J_{ij} [XYZ]_{ij}$. Here, we set $\hbar = 1$, and use the shorthands $[XY]_{ij} \equiv \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y$ and $[XYZ]_{ij} \equiv \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z$. We will assume $J_{ij} = J$ if i and j are nearest neighbors and $J_{ij} = 0$ otherwise.

Note that a single correlation operator can be obtained using a single-qubit Hamiltonian. For example in a one-dimensional (1D) qubit array, $K_2 = \sigma_2^x \sigma_1^z \sigma_3^z$ can be generated by the time evolution operator $e^{i(\pi/2)(\sigma_1^z + \sigma_2^x + \sigma_3^z)}$. However, it is not evident how to obtain a sum like $K_2 + K_3$ from the single-qubit Hamiltonian.

Most fabricated solid-state qubit systems are nanodevices, because a smaller size makes them more robust to decoherence. An example are quantum dot systems where smaller dots have larger energy-level spacing. Since with diminishing size it becomes difficult to address these devices individually, it is of interest to consider switching on/off H_0 and H_{int} independently. We will show that, by using appropriate pulse sequences, this is possible even if we start from an *always-on* Hamiltonian [11].

Ising model.— We now show how to construct the stabilizer Hamiltonian using the relation

$$\begin{aligned} e^{-i\theta \sigma_1^z \sigma_2^z} \sigma_1^x e^{i\theta \sigma_1^z \sigma_2^z} &= \cos(2\theta) \sigma_1^x + \sin(2\theta) \sigma_1^y \sigma_2^z, \\ e^{-i\theta \sigma_1^z \sigma_2^z} \sigma_1^y e^{i\theta \sigma_1^z \sigma_2^z} &= \cos(2\theta) \sigma_1^y - \sin(2\theta) \sigma_1^x \sigma_2^z. \end{aligned} \quad (3)$$

An important consequence of these equations is that, for $\theta = \pi/4$, we can increase the order of the Pauli-matrix terms as in $e^{-i\frac{\pi}{4} \sigma_1^z \sigma_2^z} \sigma_1^x e^{i\frac{\pi}{4} \sigma_1^z \sigma_2^z} = \sigma_1^y \sigma_2^z$ and $e^{-i\frac{\pi}{4} \sigma_1^z \sigma_2^z} \sigma_1^y e^{i\frac{\pi}{4} \sigma_1^z \sigma_2^z} = -\sigma_1^x \sigma_2^z$. For a 1D N -qubit chain, the starting single-qubit Hamiltonian is given by

$$H_s = \Omega(\sigma_1^y + \sum_{i=2}^{N-1} \sigma_i^x + \sigma_N^y). \quad (4)$$

By applying H_{int} as

$$e^{-i\theta \sum_{i < j} \sigma_i^z \sigma_j^z} H_s e^{i\theta \sum_{i < j} \sigma_i^z \sigma_j^z}, \quad (5)$$

we obtain the 1D stabilizer Hamiltonian H_{stab} for $\theta = \pi/4$. As an example, for $N = 3$ qubits, starting from $H_s = \Omega(\sigma_1^y + \sigma_2^x + \sigma_3^y)$ we obtain $H_{\text{stab}} = \Omega(-\sigma_1^x \sigma_2^z - \sigma_1^z \sigma_2^x \sigma_3^z - \sigma_3^x \sigma_2^z)$. If the system of N qubits has periodic boundary conditions, we start from the Hamiltonian $H_s = \Omega \sum_{i=1}^N \sigma_i^x$. Since Ising-type interaction terms commute and the time-evolution operator in (5) factorizes, this process can be straightforwardly extended to 2D and 3D qubit systems, thus realizing the universal resource discussed in the introduction. Consequently, for Ising interactions, we can construct the stabilizer Hamiltonian by switching on H_{Ising} only once.

We will now implement the above equations into a pulse sequence that is familiar in the NMR context [9]. We assume that each pulse is sufficiently strong such that interactions between qubits can be neglected during the pulse sequences. We describe the time evolution of the system by the density operator $\rho(t)$ whose time dependence is given by $\rho(t) = \exp(-iHt)\rho(0)\exp(iHt)$ for time-independent H . It is convenient to use the following schematic notation for this evolution: $\rho(0) \xrightarrow{tH} \rho(t)$. Then the process

$$\rho(0) \xrightarrow{\tau_1 H_{\text{int}}} \xrightarrow{\tau H_s} \xrightarrow{-\tau_1 H_{\text{int}}} \rho(t) \quad (6)$$

for $\tau_1 = \pi/(4J)$ corresponds to $\rho(0) \xrightarrow{\tau H_{\text{stab}}} \rho(t)$ where $t = \tau + \pi/(2J)$ and τ can be chosen arbitrarily. Note that at the physical time t the state of the system is obtained from the initial one by the time-evolution operator

$$e^{-i\tau H_{\text{stab}}} = e^{-i(\pi/4) \sum_{i < j} \sigma_i^z \sigma_j^z} e^{-i\tau H_s} e^{i(\pi/4) \sum_{i < j} \sigma_i^z \sigma_j^z}. \quad (7)$$

As illustrated by Fig. 1(a), H_{stab} becomes the effective system Hamiltonian. Its ground state is the originally prepared cluster state, which is therefore preserved.

XY model.— Next, we show how to generate the stabilizer Hamiltonian using the XY interaction, assuming that H_0 and H_{int} can be switched on/off independently.

The stabilizer Hamiltonian is formed step by step by bonding the nearest-neighbor operators. This is because the XY interactions do not commute, $[[XY]_{i-1,i}, [XY]_{i,i+1}] \neq 0$. We start from

$$\begin{aligned} e^{-i\theta[XY]_{12}} \sigma_1^x e^{i\theta[XY]_{12}} &= \cos(2\theta) \sigma_1^x - \sin(2\theta) \sigma_1^z \sigma_2^y, \\ e^{-i\theta[XY]_{12}} \sigma_1^y e^{i\theta[XY]_{12}} &= \cos(2\theta) \sigma_1^y + \sin(2\theta) \sigma_1^z \sigma_2^x, \\ e^{-i\theta[XY]_{12}} \sigma_1^z e^{i\theta[XY]_{12}} &= \cos^2(2\theta) \sigma_1^z + \sin^2(2\theta) \sigma_2^z \\ &+ \frac{1}{2} \sin(4\theta) [\sigma_1^x \sigma_2^y - \sigma_1^y \sigma_2^x]. \end{aligned} \quad (8)$$

For $\theta = \pi/4$, these transformations increase the order of the Pauli-matrix terms as $\sigma_1^x \rightarrow -\sigma_1^z \sigma_2^y$ and $\sigma_1^y \rightarrow \sigma_1^z \sigma_2^x$. For σ_1^z one obtains $\sigma_1^z \rightarrow \sigma_2^z$.

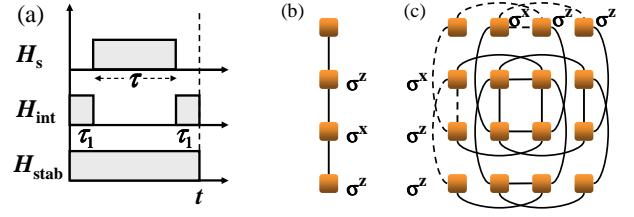


FIG. 1: (a) Switching on/off the parts H_s and H_{int} of the Hamiltonian $H = H_s + H_{\text{int}}$ gives rise to an effective time evolution described by the stabilizer Hamiltonian H_{stab} [see Eq. (7)] whose ground state is a cluster state. (b) Graph representation of a one-dimensional 4-qubit (squares) cluster state stabilized by the Ising interaction. (c) Two-dimensional 4×4 twisted cluster state stabilized by the XY interaction, a universal resource for one-way quantum computation. The dashed lines and Pauli operators in each direction illustrate the twistedness of the state and the corresponding stabilizer Hamiltonian.

We now show how to construct a 2D stabilizer Hamiltonian. First we construct the 1D stabilizer Hamiltonian, starting from

$$H_s = \Omega(-\sigma_1^x + \sigma_2^y - \sum_{i=3, N-2} \sigma_i^x + \sigma_{N-1}^y - \sigma_N^x). \quad (9)$$

In the specific case of six qubits in 1D, by applying Eq. (8) to $[XY]_{12}$, $[XY]_{34}$, and $[XY]_{56}$, we obtain:

$$\begin{aligned} e^{-iS_1} H_s e^{iS_1} &= \Omega(\sigma_1^z \sigma_2^y + \sigma_2^z \sigma_1^x + \sigma_3^z \sigma_4^y + \sigma_4^z \sigma_3^y \\ &+ \sigma_5^z \sigma_6^y + \sigma_6^z \sigma_5^y), \end{aligned} \quad (10)$$

where $S_1 = \frac{\pi}{4} \sum_{l=1,3} [XY]_{\{2l-1, 2l\}}$. Repeating this step with $S_2 = \frac{\pi}{4} \sum_{l=1,2} [XY]_{\{2l, 2l+1\}}$, we get the 1D stabilizer Hamiltonian $H_{1D} = e^{-iS_2} e^{-iS_1} H_s e^{iS_1} e^{iS_2}$ that reads explicitly

$$\begin{aligned} H_{1D} &= \Omega(\sigma_1^z \sigma_2^z \sigma_3^x + \sigma_3^z \sigma_1^x + \sigma_2^z \sigma_4^z \sigma_5^x + \sigma_5^z \sigma_3^z \sigma_2^x \\ &+ \sigma_6^z \sigma_6^x + \sigma_6^z \sigma_5^z \sigma_4^x). \end{aligned} \quad (11)$$

This Hamiltonian is twisted in the sense of [4], i.e., the site indices of the corresponding cluster state are obtained by the permutation $(2, 3)(4, 5) \dots (N-2, N-1)$ (cyclic notation), for a chain of N qubits where N is even, see Fig. 1(b,c).

The next step in the construction of the 2D stabilizer Hamiltonian is to construct a ladder Hamiltonian by bonding nearest-neighbor sites on adjacent chains a and b , in which all the bondings between qubits ia and ib are carried out simultaneously:

$$\begin{aligned} H_{\text{ladder}} &= \Omega(-\sigma_{1a}^z \sigma_{2a}^z \sigma_{3a}^z \sigma_{3a}^y - \sigma_{3a}^z \sigma_{1b}^z \sigma_{1a}^y \\ &- \sigma_{2a}^z \sigma_{4a}^z \sigma_{5b}^z \sigma_{5a}^y - \sigma_{5a}^z \sigma_{3a}^z \sigma_{2b}^z \sigma_{2a}^y - \sigma_{4a}^z \sigma_{6a}^y \sigma_{6b}^z \\ &- \sigma_{6a}^z \sigma_{5a}^z \sigma_{4b}^z \sigma_{4a}^y) + (a \leftrightarrow b). \end{aligned} \quad (12)$$

A 2D stabilizer Hamiltonian is produced by connecting the above two ladder Hamiltonians with the interaction

between the two ladders. For example, when we prepare two ladders of length 4 such as in Fig. 1(c) and connect them vertically, we obtain a 4×4 stabilizer Hamiltonian.

Heisenberg model.— For the Heisenberg interaction, we can construct only a two-qubit stabilizer Hamiltonian (note that the same is true for the XXZ interaction). The basic relation is

$$\begin{aligned} e^{-i\theta[XYZ]_{12}}\sigma_1^y e^{i\theta[XYZ]_{12}} &= \cos^2(2\theta)\sigma_1^y \\ &+ \sin^2(2\theta)\sigma_2^y + \frac{1}{2}\sin(4\theta)(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x). \end{aligned} \quad (13)$$

For the Ising and XY models, we can eliminate the single Pauli matrix terms leaving the interaction terms [see Eqs. (3) and (8)]. However, in Eq. (13), if we set $\sin(2\theta) = 0$ or $\cos(2\theta) = 0$, we also eliminate the $\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x$ term. This is because the Heisenberg interaction contains terms in all three spatial directions [4]. In the case of two qubits, we obtain $H = \Omega(\sigma_1^z\sigma_2^x - \sigma_1^x\sigma_2^z)$ from the initial Hamiltonian $H_s = \Omega(\sigma_1^y - \sigma_2^y)$ by using Eq. (13) for $\theta = \pi/8$. By applying a π -rotation, we obtain the two-qubit stabilizer Hamiltonian $H = \Omega(\sigma_1^z\sigma_2^x + \sigma_1^x\sigma_2^z)$.

Manipulation of always-on Hamiltonian.— The scheme discussed up to now relies on switching on/off the single-qubit Hamiltonian H_0 [see Eq. (2)] and the Ising or XY interaction part H_{int} separately. There is a number of schemes for switching on/off interactions between qubits (see, e.g., [12–14]). However, they make the system more complicated and require additional overhead.

Here, we solve this problem by demonstrating how to extract H_0 and H_{int} by using appropriate pulse sequences. We illustrate the idea using the standard NMR Hamiltonian $H_{\text{nmr}} = \sum_i \varepsilon_i \sigma_i^z + \sum_{i < j} J \sigma_i^z \sigma_j^z$ which has the property that $[H_0, H_{\text{int}}] = 0$. In this case, H_0 and H_{int} can be switched on/off by using a simple pulse sequence. The interaction part H_{Ising} can be extracted by using two sandwiched π -pulses such as $\exp(i\tau H_{\text{Ising}}) = e^{-i(\pi/2)} \sum_j \sigma_j^y e^{i(\pi/2)} H_{\text{nmr}} e^{i(\pi/2)} \sum_j \sigma_j^y e^{i(\tau/2)} H_{\text{nmr}}$. On the other hand, two steps are required to obtain H_0 . Let us consider a 1D qubit chain. By applying a π -pulse about the x -axis (denoted by $(\pi)_x$) to all the qubits on the even sites, we obtain $e^{-i(\pi/2)} \sum_i \sigma_{2i}^y e^{i(\tau/2)} H_{\text{nmr}} e^{i(\pi/2)} \sum_i \sigma_{2i}^y e^{i(\tau/2)} H_{\text{nmr}} = e^{i\tau} \sum_i \Omega \sigma_{2i-1}^z$. Similarly, we obtain $e^{i\tau} \sum_i \Omega \sigma_{2i-1}^z$ by applying a $(\pi)_x$ -pulse to all the qubits on the odd sites. Combining these two processes yields H_0 . This method is easily generalized to 2D or 3D qubit arrays.

If $[H_0, H_{\text{int}}] \neq 0$, this NMR method cannot be used. Even in this case, H_0 and H_{int} can be extracted separately. The idea follows from average Hamiltonian theory which is based on the Baker-Campbell-Hausdorff (BCH) formula for the expansion of $e^A e^B$ [9]. A stroboscopic application of the Hamiltonian designed by a series of short pulses can reduce or eliminate unwanted terms, if $\Omega\tau, J\tau \ll 1$. First we extract H_{int} by setting $A = h_0 + h_1$ and $B = -h_0 + h_1$ in the BCH formula, where $h_0 \equiv H_0\tau$

and $h_1 \equiv H_{\text{int}}\tau$. B is realized by applying a $(\pi)_y$ -pulse on every qubit. From the BCH formula, we obtain

$$e^A e^B \approx \exp(2h_1 + [h_0, h_1] + \frac{1}{3}[h_0, [h_0, h_1]]). \quad (14)$$

The exponent corresponds to a third-order expansion in $\Omega\tau$ for $\Omega\tau \ll 1$. If we repeat this operation n times like $e^A e^B e^A e^B \cdots e^A e^B = (e^A e^B)^n$ such that $n\Omega\tau = \pi/4$, the k -th term is of order $[\pi/(4n)]^k$. Therefore, H_0 is cancelled, and we obtain only H_{int} in this order. When we apply

$$e^A e^B e^B e^A \approx \exp(4h_1 - \frac{5}{3}[h_1, [h_0, h_1]] + \frac{1}{3}[h_0, [h_0, h_1]]), \quad (15)$$

we can eliminate the second term in Eq. (14). In the limit $n \rightarrow \infty$ under the condition of $n\Omega\tau = \pi/4$, H_0 is exactly eliminated. The extraction of H_0 can be achieved analogously. Moreover, as shown in [15], if the k th-order term is the first nonvanishing correction, the decay rate T_2 of the qubit system is enlarged according to $T'_2 \propto T_2(k+1)!(T_2/t_c)^k$ as long as $T_2 > t_c$ where t_c is the time required for each single step [A and B in Eq. (14)].

For the XY model, we have to switch off subsets of H_{int} corresponding to S_1 and S_2 , as discussed after Eq. (10). This is equivalent to choosing A and B appropriately: e.g., for the 1D chain, $A = H_{\text{int}} = h_{1e} + h_{1o}$ and $B = h_{1e} - h_{1o}$ where $h_{1e} = J\tau([XY]_{23} + [XY]_{45} + [XY]_{67} + \dots)$ and $h_{1o} = J\tau([XY]_{12} + [XY]_{34} + [XY]_{56} + \dots)$. That means, B is generated by applying π pulses to qubits 2, 3, 6, 7, 10, 11,

Let us consider the 1D XY model with $\varepsilon_i = 0$ in Eq. (2). The following operation can be used to obtain H_0 . (i) Applying a $(\pi)_x$ -pulse to all the qubits on the even sites changes the sign of $\sum_{i < j} \sigma_i^y \sigma_j^y$. (ii) By further applying a $(\pi)_y$ -pulse to the same subset of qubits, we obtain $B/\tau = \sum_i \Omega(\sigma_{2i-1}^x - \sigma_{2i}^x) - \sum_{i < j} J[XY]_{ij}$. As a result, we obtain $\Omega \sum_i \sigma_{2i-1}^x$. Repeating the same operations with the qubits on the odd sites, we obtain $\Omega \sum_i \sigma_{2i}^x$. For the Ising model, the process (i) is not required. For the Heisenberg model, the same procedure as in the case of the XY model does not eliminate the term $J \sum_{i < j} \sigma_i^z \sigma_j^z$. Thus, additional similar steps are required to eliminate this term.

Robustness.— Since a practical realization of these pulse sequences will not be free of imperfections, we now analyze the effect of pulse duration errors δ . A central quantity will be the cluster-state fidelity $F_{\text{st}}(\tau) = |\langle \Psi_{00\dots 0} | U_{\tau}(\delta) | \Psi_{00\dots 0} \rangle|^2$ where $U_{\tau}(\delta) = e^{-i\tau H_{\text{stab}}(\pi/4 + \delta/4)}$. In the Ising case, $H_{\text{stab}}(\theta)$ is given by Eq. (5), and an analogous equation in the XY case.

Let us consider the 1D Ising case. For $\theta = \pi/4 + \delta/4$, the first-order correction to Eq. (5) reads

$$H'(\delta) = \frac{\delta}{2} \Omega \left[\sigma_1^y + \sigma_N^y + \sum_{1 < i < N} \sigma_i^y (\sigma_{i-1}^z + \sigma_{i+1}^z) \right]. \quad (16)$$

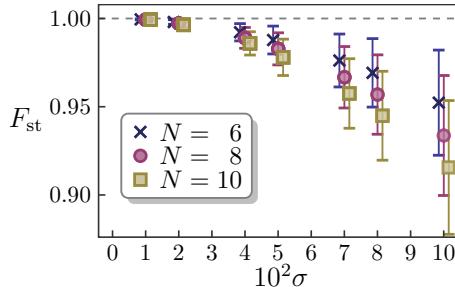


FIG. 2: Dependence of the cluster-state fidelity for $\tau = \pi/(4\Omega)$ on random errors in the pulse duration for one-dimensional XY qubit arrays of length $N = 6, 8$, and 10 . The width of the Gaussian distribution is denoted by σ ; the error bars indicate the standard deviation.

The effect of these terms is calculated from perturbation theory using the expressions for $\sigma_i^\alpha |\Psi_{00..0}\rangle$, where $|\Psi_{00..0}\rangle$ is the initial cluster state and $\alpha = x, y, z$ [16]. The lowest-order expression of the cluster-state fidelity reads $F_{\text{st}}(\tau) \approx [1 + \Omega^2 \tau^2 \delta^2 (N-1)/2]^{-2}$, and the correction scales with δ^2 which is a signature of the robustness of our method.

The simplest and most powerful method to further reduce the effect of pulse imperfections is the symmetrization of the pulse sequence frequently used in NMR [9]. We first note that Eq. (6) is equivalent to

$$\rho(0) \xrightarrow{\tau_1 H_{\text{int}}} \xrightarrow{\frac{\tau}{2} H_0} \xrightarrow{-\tau_1 H_{\text{int}}} \xrightarrow{-\tau_1 H_{\text{int}}} \xrightarrow{\frac{\tau}{2} H_0} \xrightarrow{\tau_1 H_{\text{int}}} \rho(t), \quad (17)$$

where, as before, $\tau_1 = \pi/(4J)$. The second half of this pulse sequence results in a perturbation term that has the opposite sign as compared to Eq. (16). Applying Eq. (14) leads to a cancellation of the first-order perturbation term. If the original interval length τ is divided into an even number n of subintervals, $\tau = nt_c$, the perturbation term is replaced by $[H_{\text{stab}}, H']i\tau\delta/(4n)$, and the fidelity F_{sym}^n is given by $F_{\text{sym}}^n \approx [1 + \Omega^4(2\tau^2/n)^2\delta^2(N-1)/2]^{-2}$. Hence, the fidelity is improved, $F_{\text{sym}}^n > F_{\text{st}}$, if $n > 2\Omega\tau$.

These perturbative results are complemented by exact numerical calculations of $F_{\text{st}}(\tau)$ and the gate fidelity $F_g(\tau) = 2^{-N} |\text{Tr } U_\tau^\dagger(0)U_\tau(\{\delta_i\})|$ for $\tau = \pi/(4\Omega)$ in systems with up to 10 qubits. Here, δ_i , $i = 1, \dots, N-1$ corresponds to the qubit pair $(i, i+1)$. In both the Ising and XY case, we averaged these fidelities over 2000 random realizations of the δ_i taken from a Gaussian distribution with varying width σ . The results indicate that the method is rather robust even outside the regime where σ is much smaller than $\pi/4 \approx 0.78$. For instance, the XY -model calculation shows that F_{st} is bigger than 99% for $\sigma \lesssim 0.04$, while F_g can be bigger than 99% even for σ as large as 0.07. The comparison between the two models shows that the robustness in the XY case (see Fig. 2) is somewhat better than in the Ising case.

Discussion and Conclusion.— The stabilizer formal-

ism relies on the multiplication of Pauli matrices and is widely used in quantum information theory, e.g. in quantum error-correcting codes (QECC) [17]. The distinguishing feature of the stabilizer Hamiltonians discussed here is that their ground states are directly related to the universal resource of measurement-based quantum computation. In QECC, the stabilizer formalism is used to express codewords, and the method illustrated in Eq. (6) can be used to obtain eigenvalues for the syndrome measurements in the process of detecting errors. Moreover, relations (3) and (8) can be used to effectively generate codewords in solid-state qubits. For example, the three-qubit GHZ state $(|000\rangle \pm |111\rangle)/\sqrt{2}$, which is used for the nine-qubit code, is effectively generated by $e^{\pm i(\pi/4)\sigma_1^x\sigma_2^x\sigma_3^x}|000\rangle$, where $e^{i(\pi/4)\sigma_1^x\sigma_2^x\sigma_3^x} = e^{i(\pi/2)(\sigma_1^y+\sigma_3^y)}e^{\pm i(\pi/4)\sigma_1^z\sigma_2^z\sigma_3^z}e^{-i(\pi/2)(\sigma_1^y+\sigma_3^y)}$ is applied to the three-qubit array. The five-qubit code [17] can also be generated by using this method.

To conclude, we have proposed a method to produce 2D stabilizer Hamiltonians by using pulse sequences starting from natural qubit-qubit interactions. The ground states of these Hamiltonians are 2D cluster states that are universal resources of measurement-based quantum computation. We have shown that by specially tailored pulse sequences, initially prepared cluster states can be preserved with high fidelity. We have also shown how this can be implemented in the case of always-on interactions. Our work will facilitate implementations of one-way quantum computing.

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